February 7, 2008

LBL-37893
UCB-PTH-95/35

A New Supersymmetric Framework For Fermion Masses. *

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Abstract

Supersymmetric theories involving a spontaneously broken flavor symmetry can solve the flavor-changing problem while having quark and lepton masses derived from both F and D terms. As an example, a theory of leptons is constructed in which holomorphy constrains the electron to be massless at tree level. The electron flavor symmetries are broken by D terms, leading to flavor mixing in the slepton mass matrices, which allows a radiative electron mass to be generated by the gauge interactions of supersymmetric QED. Such a radiative origin for the electron mass can be probed by searches for $\tau \to e\gamma$, and could be verified or eliminated by measurements of slepton pair production.

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

1. The standard model of particle physics gives no understanding of the pattern of quark and lepton masses and mixings; for example, why is the electron so light? The minimal supersymmetric extension of the standard model, while providing considerable insight into the origin of electroweak symmetry breaking, has made no progress whatever on this fermion mass problem. For each measured mass or mixing angle there is a corresponding Yukawa coupling, which, as in the standard model, must simply be chosen to fit the data. In this letter we propose an alternative framework for fermion masses that leads to an effective supersymmetric theory at the weak scale in which the fermion masses of the lightest generation are not described by Yukawa couplings; rather they arise as radiative effects when the superpartners are integrated out of the theory.

The puzzle of the quark and lepton masses can be described in terms of the pattern of flavor symmetry breaking. Consider the Yukawa interaction $\lambda_e \bar{e}_L e_R h$ responsible for the electron mass, which breaks the independent $U(1)$ phase rotations of e_L and e_R. The smallness of the electron mass requires λ_e to be small, implying that these flavor symmetries are only very weakly broken in nature. Why? One attractive possibility is that this flavor symmetry breaking, and therefore the electron mass itself, occurs only as a radiative correction. In the very paper of 1971 in which spontaneously broken gauge theories were shown to be renormalizable [1], it was also remarked that certain mass ratios might be generated by calculable radiative corrections. Soon afterwards, a theory was constructed in which m_e/m_μ occurred as an $O(\alpha)$ radiative effect, and it is instructive to recall two crucial aspects of this scheme [2].

1. There is a flavor symmetry, G_f, which allows only one independent Yukawa coupling. The form of this Yukawa interaction is such that, even when G_f is completely broken, a mass is generated only for one fermion, identified as the muon, while the other remains massless at tree level due to an accidental electron flavor symmetry of this Yukawa interaction.

2. Other interactions of the theory, involving new particles, break this accidental electron flavor symmetry, and appear in loop diagrams to generate $m_e/m_\mu \approx O(\alpha)$.

These requirements are easily extended to apply to any case where it is desired to obtain a fermion mass or mixing angle purely from radiative corrections.

In the model of Reference 2, G_f was obtained by extending the electroweak gauge symmetry to $SU(3)_L \times SU(3)_R$, so that electrons and muons appeared in the same irreducible multiplet. When this is broken to the usual $SU(2) \times U(1)$ electroweak symmetry, the single Yukawa coupling leads to a mass only for the muon. The electron mass is generated by a loop diagram involving the broken gauge interactions, with the heavy gauge bosons appearing in the loop. This model satisfies the above two requirements, allowing an understanding of m_e/m_μ as an $O(\alpha)$ radiative effect.

This model also serves to illustrate the difficulties which have plagued attempts to use radiative corrections to understand the fermion mass spectrum.

A) It is not easy to construct Yukawa interactions which satisfy the first requirement. In the above model it involves a special vacuum alignment, which requires a considerable complication of the theory.

B) There is very little motivation for the new flavor symmetry breaking interactions and exotic particles of the second requirement. In the above model there is a large extension of the electroweak gauge group which involves doubly charged gauge bosons and which is not easily extended to the quark sector.

C) The size of the radiative fermion mass cannot be predicted because it depends on mass ratios of the new exotic particles. Furthermore, these exotic particles may all be made arbitrarily heavy so that the scheme may not have any testable consequences.

It is perhaps for these reasons that the idea of radiative fermion masses has not been as successful as originally hoped. In this letter we argue that theories which incorporate weak scale supersymmetry possess features which allow all three of the above difficulties to be addressed:

A) The Yukawa couplings of the superpotential are not only restricted by G_f, but also by holomorphy.

B) The accidental flavor symmetries of the Yukawa interactions are typically broken by the supersymmetric gauge interactions of $SU(3) \times SU(2) \times U(1)$: there is no need to postulate any new interactions beyond those required by supersymmetry. Furthermore, the particles in the loop are just the superpartners of the known gauge bosons, quarks and leptons.

C) The hierarchy problem dictates that these superpartners are lighter than

about 1 TeV, so that a supersymmetric scheme for radiative masses necessarily leads to other testable consequences.

In this letter we outline a general framework for flavor in supersymmetric theories which follows from imposing a flavor symmetry, G_f, and a pattern for its breaking. We briefly summarize which quark and lepton masses and mixing parameters can be obtained radiatively in this framework, and which must occur at tree level. We illustrate our ideas with a few simple explicit models for the lepton sector. A further discussion of the models, and a complete discussion of the quark sector, is given in a companion article [3]. We conclude by stressing that a radiative origin for m_e can be experimentally tested at future accelerators.

2. At energy scales much larger than the scale of supersymmetry breaking $\tilde{m}$, the theory is supersymmetric and the non-renormalization theorems guarantee that the only corrections to fermion masses occur via wavefunction renormalizations. These corrections cannot give mass to a previously massless fermion, and are not important for this letter. At energy scales well beneath $\tilde{m}$, the effective theory is just that of the standard model, and radiative corrections to the fermion masses are similarly uninteresting. We are therefore interested in radiative corrections at the scale $\tilde{m}$.

In this letter, and in the companion article [3], we assume that the effective theory at scale $\tilde{m}$ has the minimal gauge group, $SU(3) \times SU(2) \times U(1)$, and the minimal supersymmetric field content, three generations of quarks and leptons and two Higgs doublets. The flavor symmetry group of the pure gauge interactions is $U(3)^5 = \prod_a U(3)_a$ $(a = q, u, d, \ell, e)$ as in the standard model, where q and ℓ are the left-handed quark and lepton doublets, while u, d and e are the right-handed quark and lepton weak singlets.

The flavor group $U(3)^5$ is broken by eleven flavor matrices. Three of these are the Yukawa matrices $\boldsymbol{\lambda}_\alpha (\alpha = u, d, e)$, familiar from the standard model, while the remaining eight matrices involve soft supersymmetry breaking interactions. These are the three matrix couplings of trilinear scalar interactions $\boldsymbol{\xi}_\alpha (\alpha = u, d, e)$, which have the same $U(3)^5$ transformation properties as the $\boldsymbol{\lambda}_\alpha$, and the five scalar mass-squared matrices $\mathbf{m}_a^2$, which transform differently. For example, while $\boldsymbol{\lambda}_e$ and $\boldsymbol{\xi}_e$ transform as (3,3) under $SU(3)_\ell \times SU(3)_e$, $\mathbf{m}_\ell^2$ transforms as 1 + 8 under $SU(3)_\ell$ and $\mathbf{m}_e^2$ as 1 + 8 under $SU(3)_e$.

There is considerable freedom in assignment of $U(3)^5$ breaking to $\boldsymbol{\lambda}_\alpha, \boldsymbol{\xi}_\alpha$ and $\mathbf{m}_a^2$. The standard viewpoint assumes that the origin of all $U(3)^5$ breaking, and therefore of all fermion masses, resides in the $\boldsymbol{\lambda}_\alpha$. The constraints from rare flavor-changing processes are satisfied by taking $\boldsymbol{\xi}_\alpha$ proportional to $\boldsymbol{\lambda}_\alpha$, and each $\mathbf{m}_a^2$ proportional to the unit matrix, so that the soft operators contain no new information about the breaking of flavor symmetries. This approach requires very large hierarchies to be built into $\boldsymbol{\lambda}_\alpha$; it also misses the opportunity to make use of the advantages, outlined earlier, that supersymmetry provides for radiative masses

We consider theories with the most general set of couplings consistent with a flavor symmetry G_f. We do not allow G_f to be an R symmetry, ensuring that $\boldsymbol{\lambda}_\alpha$ and $\boldsymbol{\xi}_\alpha$ transform identically under G_f, and hence have the same rank. We require that this rank be less than three, at least for some α. Even if $\boldsymbol{\lambda}_\alpha$ and $\boldsymbol{\xi}_\alpha$ have a zero eigenvalue, the corresponding fermion can acquire a mass from radiative corrections at scale $\tilde{m}$ [4]. At 1 loop order there is a single relevant diagram, shown in Figure 1 for the case of the leptons. Choosing a basis for the $\ell, \tilde{\ell}, e$ and $\tilde{e}$ fields such that $\boldsymbol{\lambda}_e, \mathbf{m}_\ell^2$ and $\mathbf{m}_e^2$ are all diagonal, the radiative contribution to the lepton masses involve $\mathbf{V}_\ell$ and $\mathbf{V}_e$, the $SU(3)_\ell$ and $SU(3)_e$ breaking flavor mixing matrices induced at the neutral gaugino vertices by relative rotations of fermions and scalars. They also involve the scalar trilinear vertices of strength $\boldsymbol{\xi}_e + \mu \tan\beta \boldsymbol{\lambda}_e$, which also break axial lepton number, allowing a connection between the ℓ and e sectors. Although $\boldsymbol{\lambda}_e$ and $\boldsymbol{\xi}_e$ have a zero eigenvalue, the fermion which is massless at tree level can acquire a mass via this diagram because of the mixings in $\mathbf{V}_\ell$ and $\mathbf{V}_e$. Above the weak scale, this crucial information about $SU(3)_\ell \times SU(3)_e$ breaking is encoded in $\mathbf{m}_\ell^2$ and $\mathbf{m}_e^2$. In a similarly defined basis for the quarks, the radiative contributions to the quark mass matrices involve the flavor mixing matrices at the gluino vertices: $\mathbf{V}_{q,u,d}$. It is clearly attractive to speculate that some of the smaller observed parameters of the flavor sector have their origin in these radiative corrections. For example, if the lightest generation masses all come from this effect, one might expect $m_u \approx m_d \approx (\alpha_s/\alpha) m_e$ [4]. In Reference 4 it was argued that such a radiative origin for m_d implied that, for $\tan\beta \approx 1, B_d^0 \bar{B}_d^0$ mixing would be maximal. Since we now know the mixing is not maximal, such a radiative d quark mass requires large $\tan\beta$. For large $\tan\beta$ it is well known that

there are sizable radiative contributions to m_b [5] and to other parameters [6], which can affect grand unified mass relations. We will not be concerned with such corrections in this paper, rather we are interested in studying which of the small parameters of the fermion mass sector can be understood as having an origin which is entirely radiative.

We study theories of flavor in which all dimensionless couplings are of order unity. However, as we will show shortly, not all small parameters in the fermion mass sector can be understood as being purely due to weak-scale loop factors. It is still necessary that the theory at the scale $\tilde{m}$ contain some small parameters in $\boldsymbol{\lambda}_\alpha$, $\boldsymbol{\xi}_\alpha$ and $\mathbf{m}_a^2$. These small parameters are generated by spontaneously breaking a flavor symmetry, causing mass mixing between light and heavy generations in such a way that F terms give rise to small entries in $\boldsymbol{\lambda}_\alpha$ and $\boldsymbol{\xi}_\alpha$, while D terms give small entries in $\mathbf{m}_a^2$.

At some mass scale M, much larger than $\tilde{m}$, we have a full theory of flavor in which all the dimensionless parameters are of order unity. This theory is based on some flavor symmetry group, G_f, which acts not only on the three light generations but also on vector-like generations with mass of order M. The scale of M is not important; we assume only that it is less than both the Planck scale and the messenger scale, where the superpartners first learn about supersymmetry breaking. We use only renormalizable interactions to construct the full theory at M, non-renormalizable interactions suppressed by the Planck scale do not alter our results. Below the scale M, the heavy vector generations are integrated out of the theory to give a G_f invariant effective theory. In addition to the fields present at scale $\tilde{m}$, this effective theory contains only gauge singlet flavon fields ϕ whose vevs break G_f. The scale of these vevs could be dynamically determined, for example by the evolution of the soft m_ϕ^2 parameters to negative values, and hence does not require the introduction of small parameters. In the models presented below, this typically requires the introduction of trilinear superpotential interactions involving ϕ, and is not studied in this paper. These vevs play a crucial role in the mixing of heavy and light generations.

As an example, consider a light lepton, with states ℓ and e, which is prevented by G_f invariance from coupling to the Higgs, h. Suppose, however, that heavy vector leptons, L and E which have the same gauge properties as ℓ and

e, have mass terms $[M_L \bar{L} L + M_E \bar{E} E]_F$, and a G_f invariant interaction with the Higgs $[LEh]_F$. The flavor symmetry which acts on ℓ and e is broken by vevs $\langle \phi_\ell \rangle = v_\ell$ and $\langle \phi_e \rangle = v_e$, leading to mixing of the heavy and light states via the interactions $[\ell \phi_\ell \bar{L} \ + \ e \phi_e \bar{E}]_F$. This mixing will induce a Higgs coupling to the light state which is small, of order $\epsilon_\ell \epsilon_e$, where $\epsilon_\ell = v_\ell / M_L$ and $\epsilon_e = v_e / M_E$. This mechanism for generating small parameters in $\boldsymbol{\lambda}_\alpha$ was introduced by Froggatt and Nielsen using Abelian G_f [7], and is illustrated in Figure 2. In the present work G_f is taken to be non-Abelian: placing the lightest two generations in a doublet of some non-Abelian G_f allows a solution to the supersymmetric flavor changing problem. If supersymmetry breaking spurions are inserted at any of the vertices of Figure 2, corresponding small entries for $\boldsymbol{\xi}_\alpha$ are also generated.

A crucial aspect of the above mechanism is the mixing of light and heavy states, which we could represent as D terms: $[(1/M_L) L^\dagger \phi_\ell \ell \ + \ (1/M_E) E^\dagger \phi_e e]_D$. These interactions involve heavy states and cannot appear in the effective theory beneath M. When they are integrated out of the theory they produce the effective F term: $(1/M_L M_E)[\ell \phi_\ell \; e \phi_e \; h]_F$.

We make use of a similar mass mixing effect to generate small entries in $\mathbf{m}_a^2$. Suppose that a heavy vector lepton with mass term $[M \bar{L} L]_F$ mixes via ϕ vevs with two different light states, which, for reasons that will emerge later, we call ℓ_1 and ℓ_3: $[\ell_1 \phi_3 \bar{L} \ + \ \ell_3 \phi_1 \bar{L}]_F$. In this case $\bar{L}$ acquires a Dirac mass coupling to a linear combination of (L, ℓ_1, ℓ_3), leaving the two orthogonal combinations massless. There is an important distinction between ℓ_3 and ℓ_1. A tree-level interaction with the Higgs is present for ℓ_3: $[\ell_3 e_3 h]_F$, but not for ℓ_1. This interaction could either be a tree-level Yukawa coupling of the full theory, or it could be induced in the effective theory by Froggatt-Nielsen mass mixing. Of the two orthogonal massless combinations of (L, ℓ_1, ℓ_3), one involves only (L, ℓ_1) and has no tree level Higgs coupling, we call it ℓ_e. The other is mainly ℓ_3 and does have a Higgs coupling, we call it ℓ_τ. The necessity to rotate from the flavor basis ℓ_1, ℓ_3 to the mass basis ℓ_e, ℓ_τ is shown diagrammatically in Figure 3, where integrating out $\bar{L}$ yields off-diagonal kinetic energy D terms in the flavor basis: $(1/M^2) \; [\ell_1^\dagger \phi_1 \; \phi_3^\dagger \ell_3]_D$.

What is the consequence for the scalar mass matrix $\mathbf{m}^2$ of performing this rotation from flavor to mass basis? If $\mathbf{m}^2$ was initially proportional to the unit matrix in the 3×3 space of (L, ℓ_1, ℓ_3) the rotation would have no consequence.

However, since L is not unified with ℓ_1 or ℓ_3 in an irreducible representation of G_f, m^2_{LL} and $m^2_{l_1 l_1}$ are unrelated. This is all that is required to generate an off-diagonal entry in the mass basis: $m^2_{e\tau}/m^2 \approx v_1 v_3/M^2$. Additional comparable contributions to $m^2_{e\tau}$ arise when the vertices of Figure 3 are the soft scalar trilinear interactions rather than the superpotential interactions.

Theories with heavy vector generations have long been used to generate hierarchical Yukawa couplings by "heavy-light" mixing induced by G_f breaking [7]. Perhaps the crucial new observation of this work is that in such theories "light-light" mixing is also generated when the heavy generations are integrated out. This leads to flavor breaking in $\mathbf{m}^2$ rather than in $\boldsymbol{\lambda}$, which can then generate fermion masses by weak-scale radiative corrections.

In a supersymmetric theory of flavor, where all couplings of the full theory are of order unity, the large fermion mass can arise directly from Yukawa couplings, but the smaller ones must come either from Froggatt-Nielson mass mixing, or from weak-scale radiative loops. In a perturbative theory of flavor, the top mass must come from a tree-level Yukawa coupling, but one could contemplate the b and τ masses originating from mass mixing or from loops. In this letter we are interested in the case that some of the light fermion masses occur radiatively, which we will find requires large $\tan\beta$, and hence it is reasonable for m_b and m_τ to arise from tree-level Yukawa couplings, with m_t/m_b described dominantly by $\tan\beta$.

An attractive possibility is for the heaviest generation to occur at tree level, while the lighter two generations both occur radiatively. One way of attempting this is to have G_f be an R symmetry, allowing the rank of $\boldsymbol{\xi}$ to be larger than that of $\boldsymbol{\lambda}$ [8,9]. For example, suppose that G_f requires λ_{22} to vanish, while allowing a non-zero ξ_{22}, which could appear in the diagram of Figure 1 yielding second generation masses. This would require a large value of ξ_{22}, and since λ_{22} vanishes, the true vacuum has large electric charge breaking vevs for the scalars of the second generation [10]. Theories of this sort are excluded unless it is possible to arrange for the universe to evolve to the desired, very long lived, metastable vacuum. Hence, if the only non-zero element of $\boldsymbol{\lambda}$ is λ_{33}, we limit the non-zero elements of $\boldsymbol{\xi}$ to ξ_{33}, ξ_{3i} and ξ_{i3}, where $i = 1, 2$.

It is straightforward to see that the lightest two generation masses can-

not be radiatively generated from ξ_{33} and non-trivial $\mathbf{m}^2$ matrices. The second generation masses could come from $V_{L_{23}}^T \xi_{33} V_{R_{32}}$ (although m_μ/m_τ is so large that a sufficient muon mass cannot be generated). Can the lightest generation mass now arise from $V_{L_{13}}^T \xi_{33} V_{R_{31}}$? In theories with significant flavor mixing angles at gaugino vertices, flavor-changing phenomenology requires considerable degeneracy amongst scalars of a given charge of the first two generations. In the limit that these scalars are exactly degenerate, an $SU(2)$ flavor symmetry is present and ensures that the electron is exactly massless. Allowing the non-degeneracies to be as large as flavor-changing phenomenology allows, generates values for $m_{u,d,e}$ which are well below the observed values. This is shown explicitly in reference 3, where it is also shown that non-zero values for ξ_{3i} and ξ_{i3} do not change the conclusion that *it is not possible to obtain masses for both light generations by radiative corrections.* This means that a supersymmetric theory of flavor, with minimal field content at the weak scale, must use the tree-level mass mixing mechanism to obtain mass for at least one of the light generations. If the mixing on the left and right-handed fermions for this generation are described by the parameters ϵ_ℓ and ϵ_e, then the effective Yukawa parameter is of order $\epsilon_\ell \epsilon_e$.

What about the origin of the mass of the remaining generation? For these to occur from tree-level mass mixing effects, there must be further, very small, flavor symmetry breaking parameters ϵ'_ℓ and ϵ'_e. The point of this letter is to demonstrate that there is no need for any such additional hierarchical parameters; the mass of the remaining generation can be radiative. Hence our picture of the hierarchy of the fermion masses of the three generations is:

$$m_3 \ : \ m_2 \ : \ m_1 = 1 : \epsilon_\ell \epsilon_e \ : \ \frac{\epsilon_\ell \epsilon_e}{16\pi^2}. \tag{1}$$

It is a very non-trivial aspect of the structure of supersymmetry that the parameters $\epsilon_{\ell,e}$, which break the flavor symmetries of the second generation, also appear at radiative order in the first generation masses. Below we show through explicit models how this arises in the lepton sector. In Reference 3 we extend the theory to incorporate quarks, and show that the up quark mass can easily occur radiatively, but a radiative down quark mass is only just consistent with data on $\overline{B}B$ mixing. We also find that while V_{cb} and the CP violating phase of the Kobayashi-Maskawa matrix could arise purely radiatively, it is not possible

for V_{us} and V_{ub} to both be radiative.

3. Our first model of lepton flavor is based on a flavor group $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$ acting only on the lightest two generations. The only small parameters of the theory are those which break this group, and hence it is the breaking of this group which contains the essence of lepton flavor. We later give extensions to $SU(3)_\ell \times SU(3)_e$. We consider an effective G_f invariant theory of leptons, in which the leptons have $SU(2)_\ell \times SU(2)_e$ transformation properties $\ell_3(1,1), \ell_A(2,1), e_3(1,1)$ and $e_a(1,2)$. The Higgs doublet transforms as $h(1,1)$, and there are just two gauge singlet flavons, $\phi_{\ell A}(2,1)$ and $\phi_{e_a}(1,2)$, whose vevs $\langle \phi_\ell \rangle = v_\ell(1,0)$, $\langle \phi_e \rangle = v_e(1,0)$ describe the breaking of G_f. The $U(1)_A$ charges are $+1$ for ℓ and ϕ_ℓ, -1 for e and ϕ_e, and 0 for l_3, e_3 and h. The $\phi_{\ell,e}$ vevs reduce the rank of G_f by 2, leaving $U(1)_\mu$, muon number, as an exact unbroken symmetry. In models with a radiative electron mass occurring by 13 mixing, it is necessary for the 23 mixing to be very small to avoid a disastrous rate for the rare decay $\mu \to e\gamma$. The origin of $U(1)_A$ will be discussed later.

The most general superpotential of the effective theory below M, which is quadratic in the lepton fields, is

$$W_{eff} = \lambda \ell_3 e_3 h + \frac{\lambda'}{M^2}(\ell\phi_\ell)(e\phi_e)h, \tag{2}$$

where λ and λ' are two dimensionless parameters of order unity. The absence of any further terms of higher dimensions can be traced to the fact that the only holomorphic G_f singlets involving ℓ, ϕ_ℓ, e and ϕ_e are $(\ell\phi_\ell)$ and $(e\phi_e)$. We have imposed R parity, which forbids interactions such as $\ell_3(\ell\phi_\ell)(e\phi_e)$.

This superpotential has remarkable features. In particular, it yields a tree level mass hierarchy $m_\tau : m_\mu : m_e = 1 : \epsilon_\ell\epsilon_e : 0$ where $\epsilon_\ell = v_\ell/M$ and $\epsilon_e = v_e/M$. Not only is the electron massless at tree level, but the superpotential possesses an accidental $U(1)_{\ell_1} \times U(1)_{e_1}$ symmetry, thus satisfying a general requirement for a theory with a radiative electron mass. It is holomorphy which yields the accidental electron flavor symmetries of the Yukawa interactions. Without holomorphy, $(\ell\phi_\ell^\dagger)(e\phi_e^\dagger)h$ would be allowed, and would give $m_e \approx m_\mu$. However, these electron chiral symmetries are not exact accidental symmetries of the entire effective theory, because they are broken by higher order D terms:

$$\frac{1}{M}\left[(\ell^\dagger\phi_\ell)\ell_3 + (e^\dagger\phi_e)e_3\right]_D. \tag{3}$$

In general such D terms would be present both as supersymmetric interactions which lead to e/τ wavefunction mixing, and, with the insertion of supersymmetry breaking spurion fields, as interactions which induce soft scalar masses mixing $\tilde{e}$ and $\tilde{\tau}$. In either case, the net effect is to generate 13 and 31 entries of $\mathbf{V}_\ell$ and $\mathbf{V}_e$ which are of order ϵ_ℓ and ϵ_e, respectively. The loop diagram of Figure 1 generates the radiative electron mass leading to the hierarchy of (1). The breaking of axial lepton number originates from λ_{33} and the breaking of the accidental electron flavor symmetries comes from the 31 entries of $\mathbf{V}_\ell$ and $\mathbf{V}_e$, yielding

$$m_e = \frac{\alpha}{4\pi c^2}\left(\frac{A+\mu\tan\beta}{m^2}\right) M_1\ I\left(\frac{M_1^2}{m^2}\right) V_{\ell_{31}} V_{e_{31}} m_\tau \tag{4}$$

where the scalar taus have been taken degenerate with mass m and are assumed to be much lighter than the selectrons, M_1 is the bino mass, c is the cosine of the weak mixing angle, and I is a dimensionless integral with $I(1) = 1/2$. Taking $\mu = M_1 = m$ gives $m_e = 0.5$ MeV $(A/m + \tan\beta) V_{\ell_{31}} V_{e_{31}}$. Since $V_{\ell_{31}} V_{e_{31}} \approx \epsilon_\ell \epsilon_e \approx m_\mu/m_\tau$, the electron mass is large enough only for large $(A/m + \tan\beta)$. The A parameter cannot be large enough to dominate this bracket without leading to a vacuum instability, hence we derive the prediction that tan β is large in this scheme:

$$\tan\beta \approx \frac{1}{\epsilon_\ell \epsilon_e}, \tag{5}$$

in the range of 10 - 50. The effective theory of lepton flavor defined by equations (2) and (3) has different origins for all three lepton masses, and leads to the hierarchies of (1). The $U(1)_A$ symmetry is a necessary component of G_f; without it D terms, like those of (3) but with $\phi_{\ell,e} \rightarrow \phi^\dagger_{\ell,e}$, would occur, giving rise to an unacceptable rate for $\mu \rightarrow e\gamma$. To avoid this, $U(1)_A$ should act as $L_e + L_\mu$ on lepton fields, and identically on ϕ_ℓ and ℓ, and on ϕ_e and e.

It is very straightforward to write down the full $SU(2)_\ell \times SU(2)_e \times U(1)_A$ invariant theory which leads to the effective theory of equations (2) and (3). The interaction of (2) which leads to the muon mass is obtained by integrating out a heavy vector lepton $L_3, \overline{L}_3, E_3, \overline{E}_3$ which is singlet under $SU(2)_\ell \times SU(2)_e$ but has $U(1)_A$ charges of $+2$ and -2 for L_3 and E_3. The superpotential is

$$\begin{aligned} W_1 \ &= \lambda \ell_3 e_3 h + M_{L_3} \overline{L}_3 L_3 + M_{E_3} \overline{E}_3 E_3 \\ &\quad + \lambda_L L_3 E_3 h + \lambda_\ell (\ell\phi_\ell) \overline{L}_3 + \lambda_e (e\phi_e) \overline{E}_3. \end{aligned} \tag{6}$$

The muon mass is generated by the Froggatt-Nielsen mass mixing diagram of Figure 2. The D terms of equation (3) are obtained by integrating out a heavy vector lepton which has L and $\overline{L}$ transform as (2,1) and E and $\overline{E}$ transform as (1, 2) under $SU(2)_\ell \times SU(2)_e$. Under $U(1)_A$, L and E transform as $+1$ and -1, so the additional interactions are

$$\begin{aligned} W_2 \; &= M_L \overline{L} L + M_E \overline{E} E + \lambda'_\ell (\overline{L} \ell) S + \lambda''_\ell (\overline{L} \phi_\ell) \ell_3 \\ &\quad + \lambda'_e (\overline{E} e) S + \lambda''_e (\overline{E} \phi_e) e_3 \end{aligned} \tag{7}$$

where S is a singlet and brackets such as $(\overline{L}\ell)$ denote an $SU(2)$ singlet combination of two doublets. The D terms which induce 13 mixing and lead to the electron mass are shown in Figure 3 for the ℓ sector (with ϕ_3 identified as S).

W_1 possesses an accidental flavor symmetry on the electron, because by holomorphy the electron only enters in the combinations $(\ell\phi_\ell)$ and $(e\phi_e)$. This accidental symmetry is broken in W_2 by the appearance of both $(\overline{L}\ell)$ and $(\overline{L}\phi_\ell)$ invariants. Nevertheless W_2 does not lead to a tree level electron mass since it does not contain the Higgs field. Adding higher dimension operators, scaled by powers of $(1/M_{Pl})$, does not alter this argument.

While the $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$ models described above provide a very simple explicit model to illustrate the origin of muon and electron masses, the passage from $SU(2)$ to $SU(3)$ allows a great simplification in the representation and Yukawa parameter structure, and also sheds light on the origin of $U(1)_A$ which leads to unbroken muon number.

The representations of the $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$ theory described above strongly suggest an underlying $SU(3)_\ell \times SU(3)_e$ structure, since they can be grouped together in complete $SU(3)$ multiplets:

$$\begin{aligned} &(3,1): \; \begin{pmatrix} \ell \\ \ell_3 \end{pmatrix}, \begin{pmatrix} \phi_\ell \\ \phi_{\ell_3} \end{pmatrix}, \begin{pmatrix} \overline{L} \\ \overline{L}_3 \end{pmatrix}; \;\; (\overline{3},1): \; \begin{pmatrix} L \\ L_3 \end{pmatrix} \\ &(1,3): \; \begin{pmatrix} e \\ e_3 \end{pmatrix}, \begin{pmatrix} \phi_e \\ \phi_{e_3} \end{pmatrix}, \begin{pmatrix} \overline{E} \\ \overline{E}_3 \end{pmatrix}; \;\; (1,\overline{3}): \; \begin{pmatrix} E \\ E_3 \end{pmatrix} \end{aligned} \tag{8}$$

with the singlet field S of (7) becoming ϕ_{ℓ_3} in the ℓ sector and ϕ_{e_3} in the e sector. The ten interactions of $W_1 + W_2$ which do not involve the Higgs doublet h, can be written in terms of four $SU(3)_\ell \times SU(3)_e$ invariants:

$$W_3 = M_L \overline{L} L + M_E \overline{E} E + \lambda_\ell (\ell \phi_\ell \overline{L}) + \lambda_e (e \phi_e \overline{E}) \tag{9}$$

where all fields are now $SU(3)$ $\underline{3}$ or $\underline{3}^*$. The $\overline{L}L$ mass term gives a degenerate mass to both the $SU(2)$ doublet and singlet heavy lepton. The $\lambda_\ell(\ell\phi_\ell\overline{L})$ interaction, which involves an $SU(3)$ epsilon symbol, incorporates all three of the interactions $\lambda_\ell, \lambda'_\ell$ and λ''_ℓ occurring in (6) and (7). The passage from $SU(2)$ to $SU(3)$ flavor symmetries therefore yields a unification of the mechanisms for the origin of m_μ and m_e. The exchange of the $SU(2)$ singlet heavy lepton which generates m_μ is partnered by the exchange of the heavy $SU(2)$ doublet lepton which generates m^2_{13}, which leads to m_e.

The unifications of the mass mixings with heavy leptons required for m_μ and m_e generation suggests that a true prediction for m_e/m_μ might be possible. We have been unable to accomplish this because the diagrams for m_μ and m_e involve different couplings to the Higgs boson h. While we find W_3 to be a convincing set of interactions to describe the mixing of heavy and light leptons, it is incomplete for a theory of flavor with $G_f = SU(3)_\ell \times SU(3)_e$ for two reasons:

1. There must be further $SU(3)_\ell \times SU(3)_e$ interactions which involve the Higgs doublet h. These should lead to the interactions $\lambda \ell_3 e_3 h + \lambda_L L_3 E_3 h$ of equation (6).

2. The two flavor multiplets, $\phi_\ell(3,1)$ and $\phi_e(1,3)$ of equation (8), are insufficient to break $SU(3)_\ell \times SU(3)_e$. We have assumed these fields to have vevs $(v_1, 0, v_3)_{\ell,e}$; but $SU(3)$ rotations could put these into the form $(0,0,v_3)_{\ell,e}$. It is necessary to introduce further flavons which have vevs which serve to define the third direction.

There are many ways to satisfy the above, depending on the $SU(3)_\ell \times SU(3)_e$ transformation properties chosen for the Higgs doublet and for the additional flavons, and below we give a straightforward example. An effective theory which accomplishes points 1 and 2 is obtained by adding flavons $\Phi(3,3)$ and $\overline{\Phi}(\overline{3},\overline{3})$ with vevs Φ_{33} and $\overline{\Phi}_{33}$ being non-zero. Keeping the Higgs doublet h as a singlet, $(1,1)$, two effective interactions can be written

$$W_4 = \frac{\tilde{\lambda}}{M}(\ell\overline{\Phi}e)h + \frac{\tilde{\lambda}_L}{M}(L\Phi E)h. \tag{10}$$

Inserting Φ and $\overline{\Phi}$ vevs, the $\tilde{\lambda}$ and $\tilde{\lambda}_L$ interactions generate the required λ and λ_L interactions of (6).

Finally we wish to give the full theory behind (10): what heavy particles

of mass M must be introduced? The simplest possibility is that there are extra heavy pairs of Higgs doublets. In addition to (8), the fields of the full theory are:

$$\begin{aligned} &(3,3) : \Phi, H, \overline{H}' \qquad (\overline{3},\overline{3}) : \overline{\Phi}, \overline{H}, H' \\ &(1,1) : h, \overline{h}. \end{aligned} \tag{11}$$

Here H and H' have the same gauge quantum numbers as h, and $\overline{H}$ and $\overline{H}'$ the same as $\overline{h}$. The interactions beyond W_3 are

$$\begin{aligned} W_5 \; &= M_H \overline{H} H + M_{H'} \overline{H}' H' + \lambda_E L E H + \lambda_e \ell e H' \\ &+ \lambda_H \overline{H} \Phi h + \lambda'_H \overline{H}' \overline{\Phi} h \end{aligned} \tag{12}$$

as well as unimportant couplings involving $\overline{h}$ and trilinear $\overline{H}' \Phi H$ type couplings. On integrating out the heavy Higgs H and H', the interactions of (12) generate the effective interactions of (10), as shown in Figure 4. This illustrates how Froggatt-Nielsen mass mixing can occur in the Higgs sector.

In our view, the generation of $\ell_3 e_3 h$ and LEh from (12) is not as elegant as the lepton mass mixing for m_e and m_μ induced by (9). Nevertheless the complete theory with fields (8) + (11) and interactions $W_3 + W_5$ allows us to address two further questions: the origin of $U(1)_A$ and of R parity.

We take the full theory to have the most general set of interactions amongst the fields of (8) + (11) which are invariant under $G_f = SU(3)_\ell \times SU(3)_e$. There are four holomorphic, G_f invariants involving the fields of (8), as shown in (9). Any higher dimension operator would just involve products of these. These interactions possess an accidental $U(1)_\ell \times U(1)_e$ symmetry where, under $U(1)_\ell, \ell$ and ϕ_ℓ have charge +1, and $\overline{L}$ and L have charge −2 and +2. Similarly, under $U(1)_e, e$ and ϕ_e have charge +1 and $\overline{E}$ and E have charge −2 and +2. (Combining the epsilon symbols with $\overline{L}$ and with $\overline{E}$, these symmetries can be understood as trialities and are $U(1)$s contained in $U(3)_\ell \times U(3)_e$). When the Higgs multiplets are added, the only interactions of (12) which break these accidental symmetries are the ones involving the leptons. At the renormalizable level, these all have the form "$\ell e h$" involving one "ℓ" and one "e". Hence, these break $U(1)_\ell \times U(1)_e$ to the axial combination, which is just lepton number on the lepton fields. After further breaking, this becomes precisely $U(1)_A$ on the

fields of the effective theory below M, leading to muon number conservation. In the context of $SU(3)$ flavor symmetries, we see that the conservation of muon number is not an ad hoc constraint placed on the theory, but arises as an automatic consequence of the simple theory of equation (9).

In the minimal supersymmetric standard model, and also in our $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$ model, R parity must be imposed by hand. However, since R parity acts on lepton fields it should surely be understood from the flavor symmetry. In the $SU(3)_\ell \times SU(3)_e$ model of $W_3 + W_5$, R parity is an accidental symmetry. In the minimal supersymmetric theories, and also in the $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$ theory, there is no symmetry distinction between ℓ and h: R parity must be imposed by hand to provide an artificial distinction to avoid too much lepton number violation. However, in the $G_f = SU(3)_\ell \times SU(3)_e$ model, there is a distinction built into the G_f structure, with ℓ transforming as (3, 1) and h as (1, 1). Even allowing for mass mixing; the heavy leptons L transform as $(\overline{3}, 1)$ and the heavy Higgs H as $(3, 3)$, maintaining sufficient difference between lepton and Higgs sectors that R parity violating couplings are all forbidden by G_f at the renormalizable level.

We have taken the theory at M to be renormalizable, however, G_f invariant non-renormalizable operators scaled by powers of M_{Pl} are to be expected. In the $SU(3)_l \times SU(3)_e$ model, these will lead to small violations of muon number and R parity, suppressed by powers of (M/M_{Pl}). Alternatively, $U(1)_A$ could be promoted to an exact symmetry by extending G_f to $U(3)_\ell \times U(3)_e$.

4. In this letter we have proposed a new framework for understanding flavor in supersymmetric theories which have a flavor symmetry G_f spontaneously broken by a set of vevs $\langle\phi\rangle$. When heavy vector generations of mass M are integrated out of the theory, G_f breaking interactions are generated which depend on the set of small parameters $\epsilon = \langle\phi\rangle / M$: scalar masses from D terms and Yukawa couplings from F terms. We have constructed theories of the lepton sector where these Yukawa couplings lead to a muon mass of order $\epsilon^2\ m_\tau$, but, because the F terms are holomorphic, the electron remains massless at tree level. The G_f breaking scalar masses lead, via the diagram of Figure 1, to a radiative electron mass of order $\epsilon^2\ m_\tau/16\pi^2$.

We find such a theory of lepton flavor, for example the one defined by

equations (2) and (3), to be simple and plausible. If superpartners are discovered, the proposed radiative mechanism for the electron mass can be tested quantitatively. The flavor-violating neutralino mixing matrix entries $V_{\ell_{31}}$ and $V_{e_{31}}$, together with the superpartner spectrum, could be measured in the reactions $e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-, \tilde{\tau}^+\tilde{e}^-, \tilde{e}^+\tilde{\tau}^-, \tilde{e}^+\tilde{e}^-$, as will be demonstrated elsewhere. Furthermore, the framework predicts large $\tan\beta$ and an observable decay rate for $\tau \rightarrow e\gamma$ [3].

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Figure Captions

1. A radiative diagram for lepton masses involving internal superpartners.

2. Mass mixing from flavon vevs, $\langle \phi_{l,e} \rangle$, induces a Higgs coupling to a light lepton.

3. Mass mixing from flavon vevs, $\langle \phi_{1,3} \rangle$, induces a flavor changing D term for the light leptons.

4. Mass mixing from flavon vev, $\left\langle \bar{\Phi} \right\rangle$, induces a tau Yukawa coupling in the $SU(3)_\ell \times SU(3)_e$ model.

Fig. 1

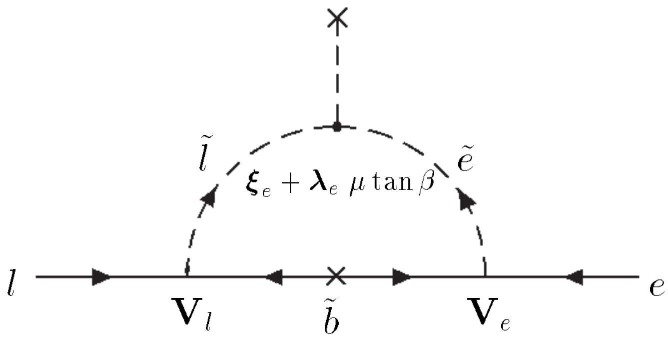

Fig. 2

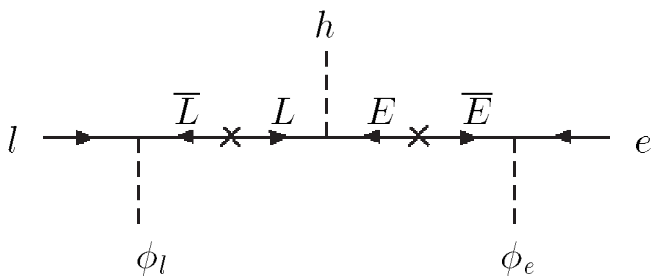

Fig. 3

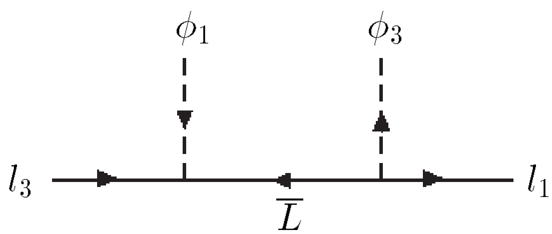

Fig. 4